Goal Programming

- **Firms usually have more than one goal. For example,**
  - maximizing total profit,
  - maximizing market share,
  - maintaining full employment,
  - providing quality ecological management,
  - minimizing noise level in the neighborhood, and
  - meeting numerous other non-economic goals.

Goal Programming

- **It is not possible for LP to have *multiple goals* unless they are all measured in the same units (such as dollars),**
  - a highly unusual situation.
- **An important technique that has been developed to supplement LP is called *goal programming*.**

Goal Programming (continued)

- Goal programming "satisfices,"
  - as opposed to LP, which tries to "optimize."
  - Satisfice means coming as close as possible to reaching goals.
- The objective function is the main difference between Goal Programming and Linear Programming.
- In Goal programming, the purpose is to minimize deviational variables,
  - which are the only terms in the objective function.

Harrison Electric Company

- The Company produces two products popular with home renovators: old-fashioned chandeliers and ceiling fans.
- Both the chandeliers and fans require a two-step production process involving wiring and assembly.
- It takes about 2 hours to wire each chandelier and 3 hours to wire a ceiling fan. Final assembly of the chandeliers and fans requires 6 and 5 hours, respectively.
- The production capability is such that only 12 hours of wiring time and 30 hours of assembly time are available.

Classical Linear Programming Formulation

If each chandelier produced nets the firm \$7 and each fan \$6, Harrison's production mix decision can be formulated using LP as follows:

$$\text{maximize profit} = \$7X_1 + \$6X_2$$

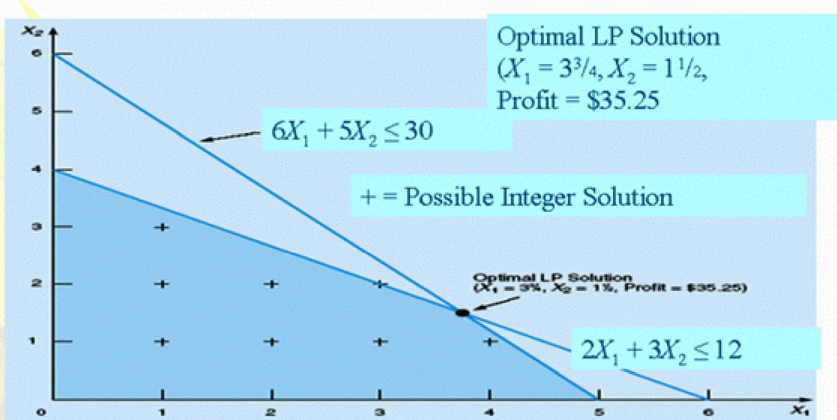
$$\begin{aligned} \text{subject to:} \quad & 2X_1 + 3X_2 \le 12 \text{ (wiring hours)} \\ & 6X_1 + 5X_2 \le 30 \text{ (assembly hours)} \\ & X_1, X_2 \ge 0 \text{ (nonnegative)} \end{aligned}$$

X_1 = number of chandeliers produced

X_2 = number of ceiling fans produced

Graphical Solution

With only two variables and two constraints, the graphical LP approach to generate the optimal solution is given below:



Example of Goal Programming
Harrison Electric Revisited

Goals Harrison's management wants to achieve, each equal in priority:

- *Goal 1:* to produce as much profit above $30 as possible during the production period.
- *Goal 2:* to fully utilize the available wiring department hours.
- *Goal 3:* to avoid overtime in the assembly department.
- *Goal 4:* to meet a contract requirement to produce at least seven ceiling fans.

Ranking Goals with Priority Levels

A key idea in goal programming is that one goal is more important than another. Priorities are assigned to each deviational variable.

GOAL	PRIORITY
Reach a profit as much above $30 as possible	P_1
Fully use wiring department hours available	P_2
Avoid assembly department overtime	P_3
Produce at least seven ceiling fans	P_4

Priority 1 is infinitely more important than Priority 2, which is infinitely more important than the next goal, and so on.

Goal 1: to produce as much profit above \$30 as possible during the production period.

Profit Goal is converted to constraint

$$7 X1 + 6 X2 >= 30$$

Let's define d_1 as deviation of LHS from \$30

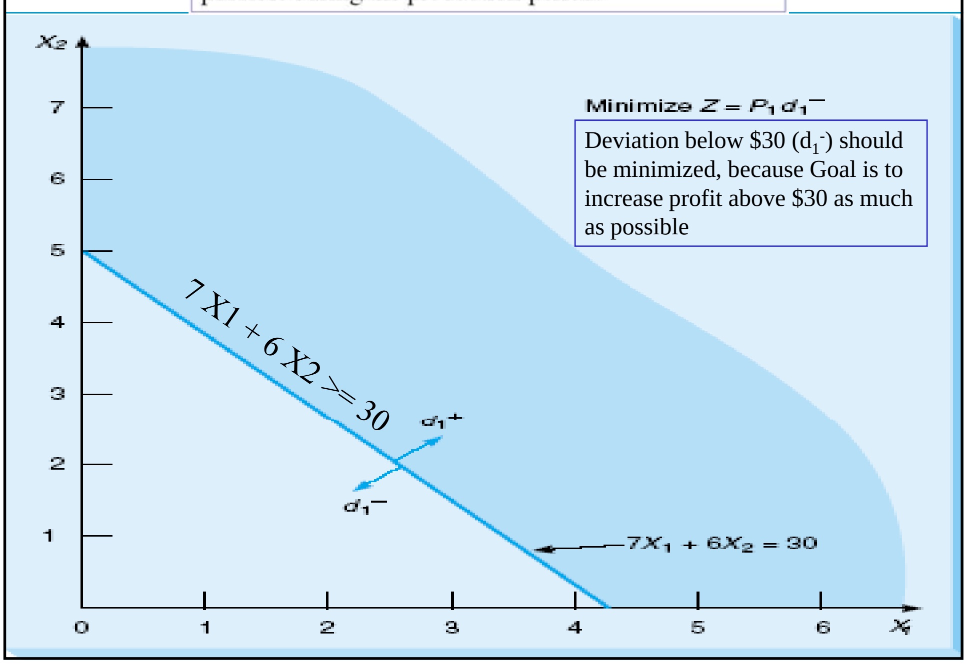
$$d_1 = (7 X1 + 6 X2) – 30$$

The deviation d_1 can be either +ve or –ve denoted by d_1^+ or d_1^-

$$d_1^+ - d_1^- = 7 X1 + 6 X2 – 30$$

The region specified by first goal is shown on graph

Goal 1: to produce as much profit above \$30 as possible during the production period.



Goal 2: to fully utilize the available wiring department hours.

Wiring department has 12 hours. According to Goal 2 , all these hours should be fully utilized. Hence the implied constraint is

$$2 X1 + 3 X2 >= 12$$

Let's define d_2 as deviation of LHS from 12 hours

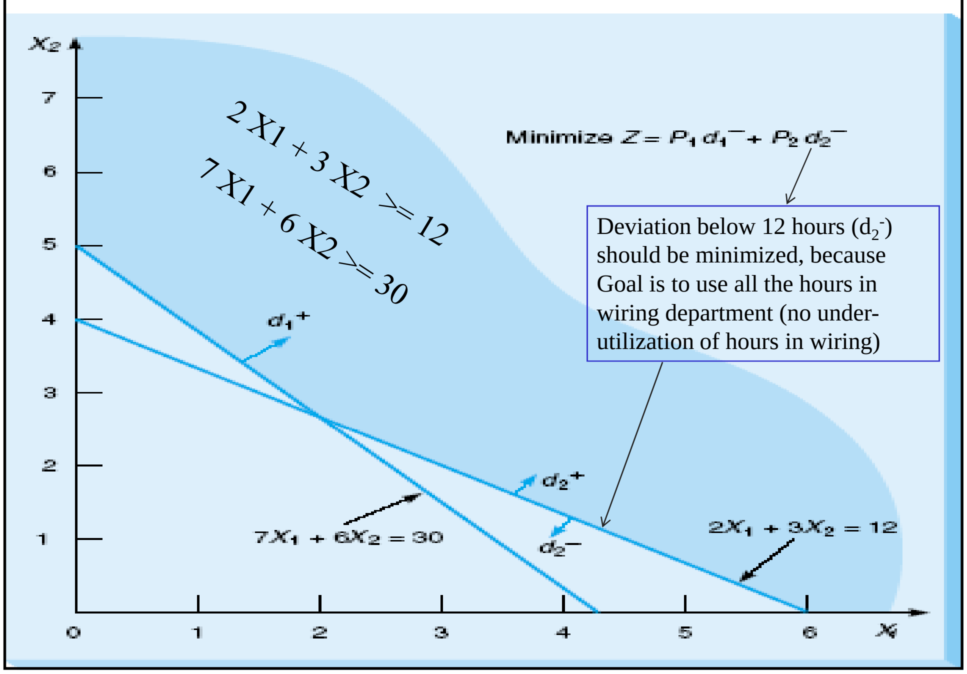
$$d_2 = (2 X1 + 3 X2) – 12$$

The deviation d_2 can be either +ve or –ve denoted by d_2^+ or d_2^-

$$d_2^+ - d_2^- = 2 X1 + 3 X2 – 12$$

The region specified by 2nd goal is shown on graph

Analysis of First and Second Goals



Goal 3: to avoid overtime in the assembly department.

Assembly department has 30 hours scheduled
According to Goal 3 , the consumed hours should not exceed 30.
Hence the implied constraint is

$$6 X1 + 5 X2 <= 30$$

Let's define d_3 as deviation of LHS from 30 hours

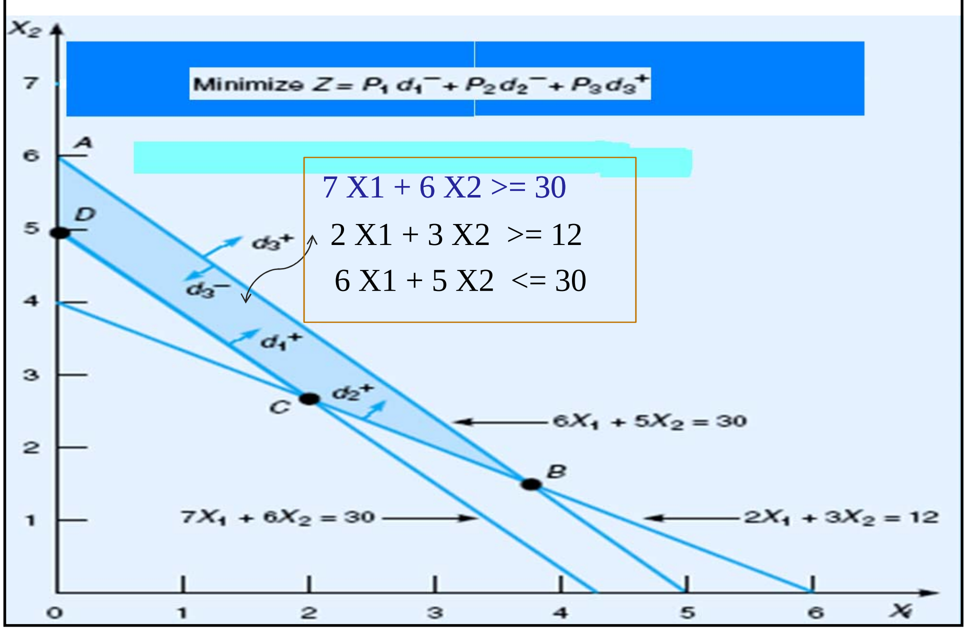
$$d_3 = (6 X1 + 5 X2) – 30$$

The deviation d_3 can be either +ve or –ve denoted by d_3^+ or d_3^-

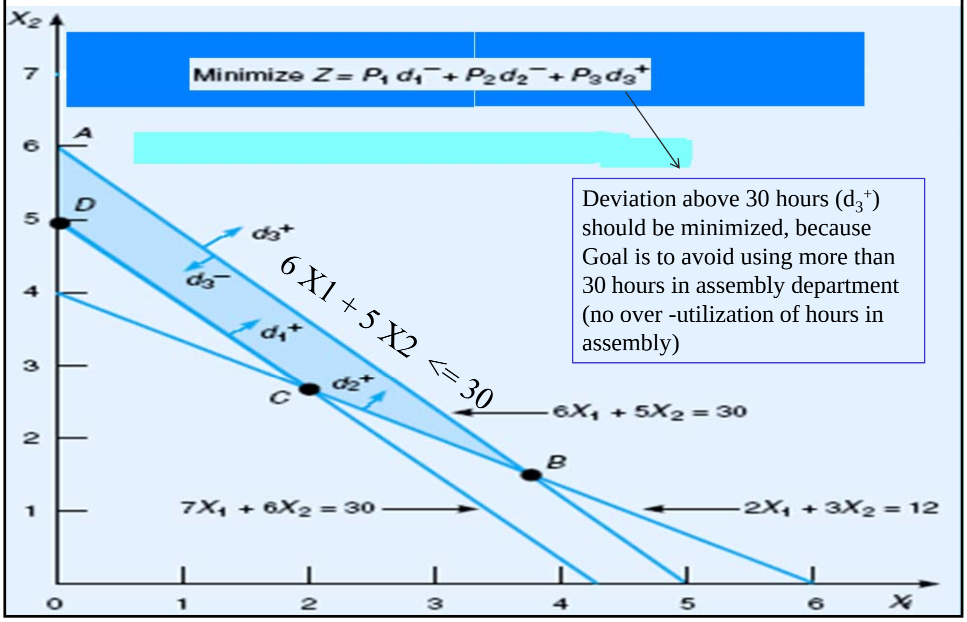
$$d_3^+ - d_3^- = 6 X1 + 5 X2 – 30$$

The region specified by 3rd goal is shown on graph

Goal 3: to avoid overtime in the assembly department.



Goal 3: to avoid overtime in the assembly department.



Goal 4: to meet a contract requirement to produce at least seven ceiling fans.

Assembly department has 30 hours scheduled
According to Goal 4 , the ceiling fans minimum quantity
Should be 7. Hence the implied constraint is

$$X2 >= 7$$

Let's define d_4 as deviation of LHS 7 units of X2

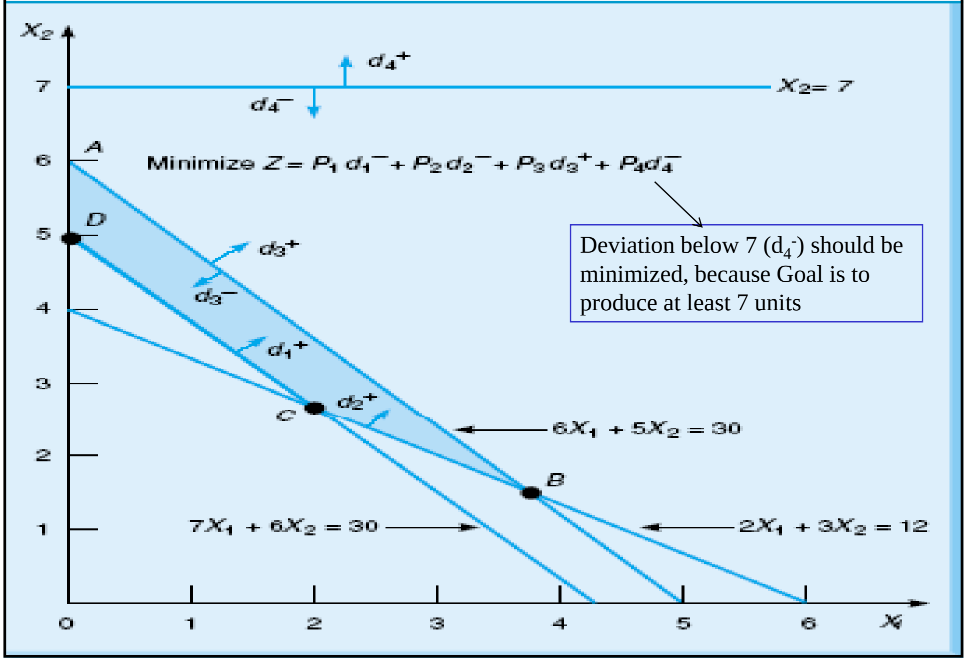
$$d_4 = X2 - 7$$

The deviation d_4 can be either +ve or –ve denoted by d_4^+ or d_4^-

$$d_4^+ - d_4^- = X2 - 7$$

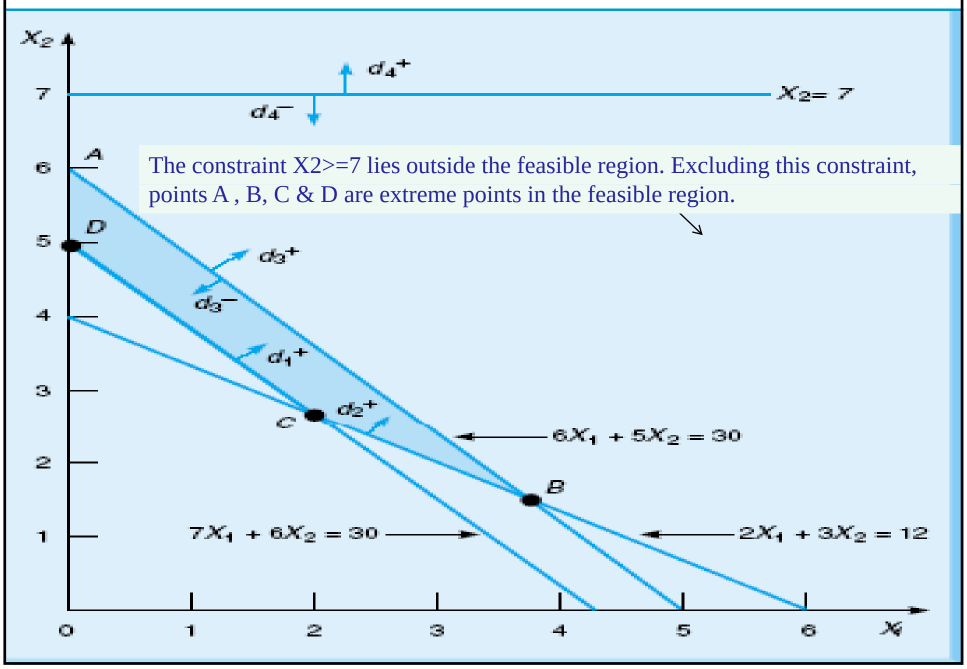
The region specified by 4th goal is shown on graph

Analysis of All Four Priority Goals



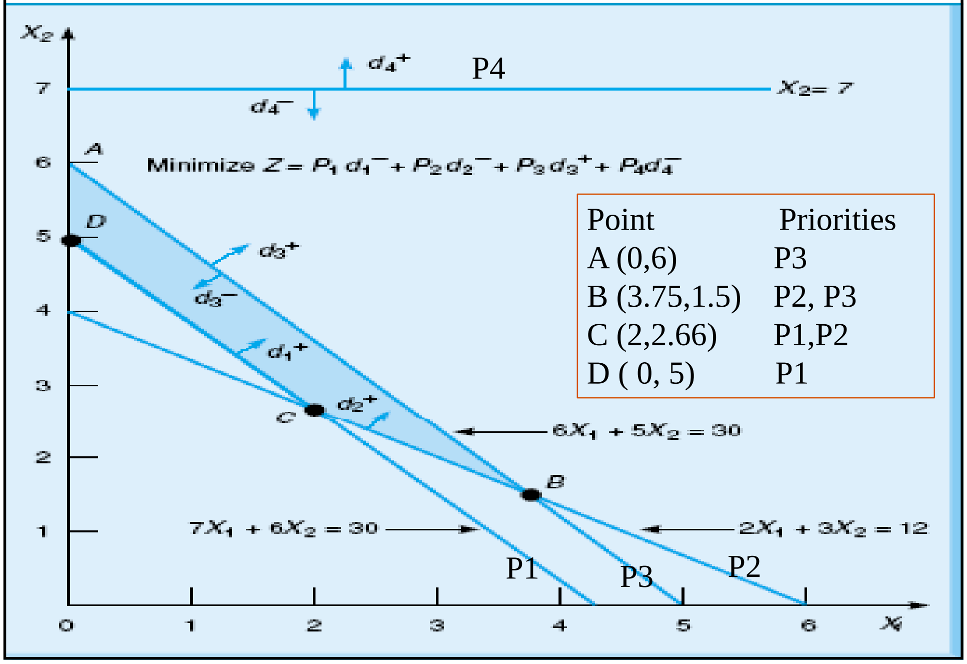
Deviation below 7 (d_4^-) should be minimized, because Goal is to produce at least 7 units

Analysis of All Four Priority Goals



The constraint X2>=7 lies outside the feasible region. Excluding this constraint, points A , B, C & D are extreme points in the feasible region.

Analysis of All Four Priority Goals



Solution at point A (0,6); priority P3

- **Goal 1 (Priority 1) : Profit should exceed $30**
-
- **7 X1 + 6 X2 = 7 (0) + 6 (6) = 36, Profit exceeds by $6**
- **Goal 2 : (Priority 2) Fully utilize wiring department's hours of 12**
-
- **2 X1 + 3 X2 = 2 (0) + 3 (6) = 18,**
- **wiring dept's utilization exceeds by 6 hours of minimum value of 12**
- **Goal 3 (Priority 3) : Avoid using more than 30 hours in assembly dept**
- **6 X1 + 5 X2 = 6 (0) + 5 (6) = 30, (zero overtime)**
- **Goal 4 (Priority 4) : Make at least 7 ceiling fans; X2 >= 7**
- **X2 = 6 (does not achieve this goal)**

Solution at point B (3.75, 1.5); priority P2,P3

- Goal 1 (Priority 1) : Profit should exceed \$30
-
- $7\,X1 + 6\,X2 = 7\,(3.75) + 6\,(1.5) = 35.3$, Profit exceeds by \$5.3
- Goal 2 : (Priority 2) Fully utilize wiring department's hours of 12
-
- $2\,X1 + 3\,X2 = 2\,(3.75) + 3\,(1.5) = 12$,
- wiring dept's utilization exceeds by 0 hours of minimum value of 12
- Goal 3 (Priority 3) : Avoid using more than 30 hours in assembly dept
- $6\,X1 + 5\,X2 = 6\,(3.75) + 5\,(1.5) = 30$, (zero overtime)
- Goal 4 (Priority 4) : Make at least 7 ceiling fans; $X2 >= 7$
- $X2 = 1.5$ (Goal falls short by 5.5 units)

Solution at point C (2, 2 2/3); priority P1,P2

- Goal 1 (Priority 1) : Profit should exceed \$30
-
- $7\,X1 + 6\,X2 = 7\,(2) + 6\,(2\ 2/3) = 0$, Profit exceeds by \$ 0
- Goal 2 : (Priority 2) Fully utilize wiring department's hours of 12
-
- $2\,X1 + 3\,X2 = 2\,(2) + 3\,(2\ 2/3) = 12$,
- wiring dept's utilization exceeds by 0 hours of minimum value of 12
- Goal 3 (Priority 3) : Avoid using more than 30 hours in assembly dept
- $6\,X1 + 5\,X2 = 6\,(2) + 5\,(2\ 2/3) = 25\ 1/3$, (4 2/3 hours are left unutilized)
- Goal 4 (Priority 4) : Make at least 7 ceiling fans; $X2 >= 7$
- $X2 = 2\ 2/3$ (Goal falls short by 4 1/3 units)

Solution at point D (0, 5); priority P1

- **Goal 1 (Priority 1) : Profit should exceed $30**
-
- **7 X1 + 6 X2 = 7 (0) + 6 (5^{3}) = 30, Profit exceeds by $ 0**
- **Goal 2 : (Priority 2) Fully utilize wiring department's hours of 12**
-
- **2 X1 + 3 X2 = 2 (0) + 3 (5) = 15,**
- **wiring dept's utilization exceeds by 3 hours of minimum value of 12**
- **Goal 3 (Priority 3) : Avoid using more than 30 hours in assembly dept**
- **6 X1 + 5 X2 = 6 (0) + 5 (5) = 25 , (5 hours are left unutilized)**
- **Goal 4 (Priority 4) : Make at least 7 ceiling fans; X2 >= 7**
- **X2 = 5 (Goal falls short by 2 units)**

Linear Programming Formulation of Goal Program

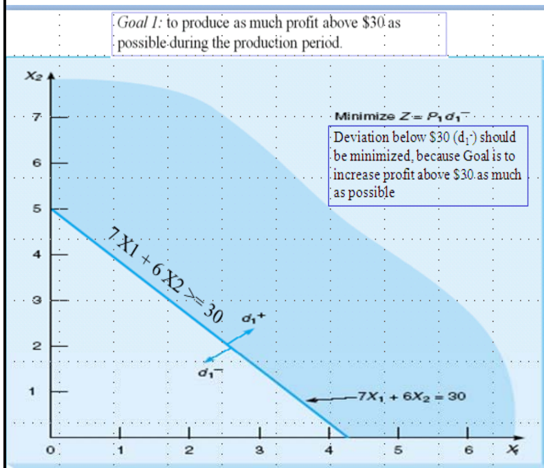
Step 1 :

Define objective function in terms of Priority values of each Goal and its corresponding deviations from the Goal

Step 2 :

Define constraint of the respective goal with upper and lower deviation values

Linear Programming Formulation of Goal Program



Goal 1:

Priority: P1

Negative deviation d_1^- should be minimum

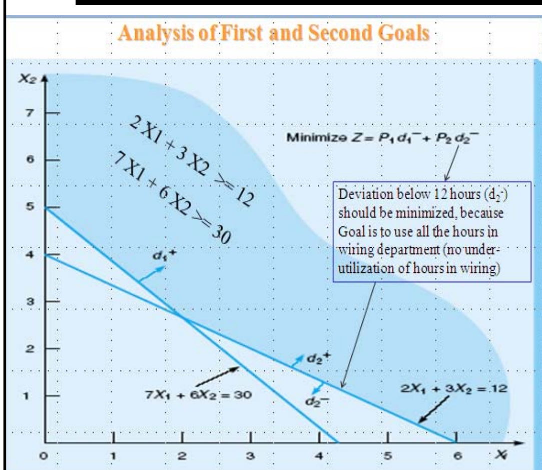
Obj function:

P1 d_1^-

Related constraint:

$d_1^+ - d_1^- = 7\,X1 + 6\,X2 - 30$

Linear Programming Formulation of Goal Program



Goal 2:

Priority: P2

Negative deviation d_2^- should be minimum

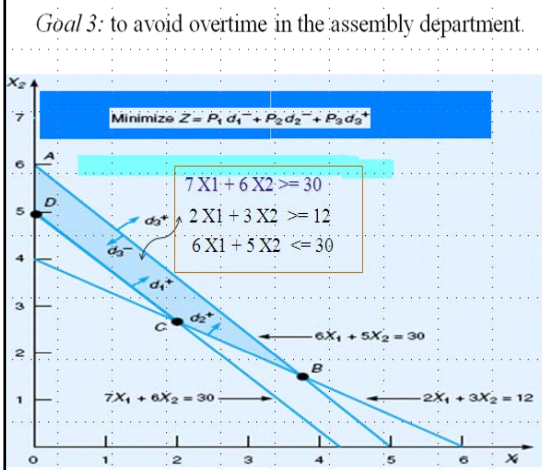
Obj function:

P2 d_2^-

Related constraint:

$d_2^+ - d_2^- = 2\,X1 + 3\,X2 - 12$

Linear Programming Formulation of Goal Program



Goal 3:

Priority: P3

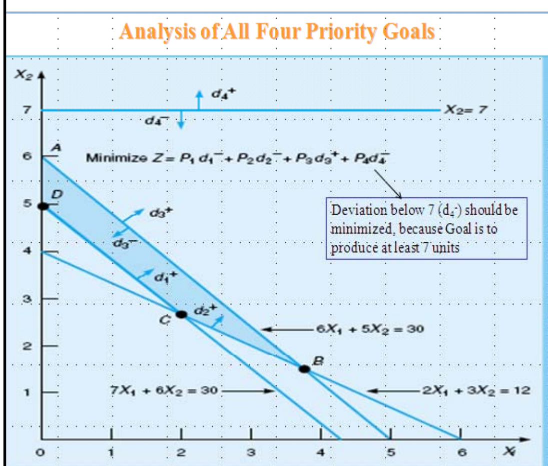
Positive deviation d_3^+ should be minimum

Objective function:

P3 d_3^+

Related constraint:
$d_3^+ - d_3^- = 6\,X1 + 5\,X2 - 30$

Linear Programming Formulation of Goal Program



Goal 4:

Priority: P4

Negative deviation d_4^- should be minimum

Objective function:

P4 d_4^-

Related constraint:
$d_4^+ - d_4^- = X2 - 7$

Linear Programming Formulation of Goal Program

Objective function:

{ Minimize prioritized deviations of all Goals }

$$\text{Min} \quad P1\, d_1^- + P2\, d_2^- + P3\, d_3^+ + P4\, d_4^-$$

subject to

{ profit should exceed $30 }

$$7\,X1 + 6\,X2 - 30 = d_1^+ - d_1^-$$

{ wiring hours should be fully utilized}

$$2\,X1 + 3\,X2 - 12 = d_2^+ - d_2^-$$

{ assembly hours should not exceed 30 hours }

$$6\,X1 + 5\,X2 - 30 = d_3^+ - d_3^-$$

{ x2 should be minimum 7}

$$X2 - 7 = d_4^+ - d_4^-$$

All variables are non-negative

Solution Methodology (Ist Obj Function)

Solve the LP model according to Priority values in the objective function;

Hence first LP model is;

$$\text{Min} \quad d_1^-$$

subject to

$$7\,X1 + 6\,X2 - 30 = d_1^+ - d_1^-$$
$$2\,X1 + 3\,X2 - 12 = d_2^+ - d_2^-$$
$$6\,X1 + 5\,X2 - 30 = d_3^+ - d_3^-$$
$$X2 - 7 = d_4^+ - d_4^-$$

Solution Methodology (2nd Obj Function)

Solution of Ist LP model will yield a value for variable d_1^-
say ; $d_1^- = K1$

Now add the value of this variable in constraint set and solve for second priority variable as follows;

Min d_2^-

subject to

$$\begin{aligned}
7\,X1 + 6\,X2 - 30 &= d_1^+ - d_1^- \\
2\,X1 + 3\,X2 - 12 &= d_2^+ - d_2^- \\
6\,X1 + 5\,X2 - 30 &= d_3^+ - d_3^- \\
X2 - 7 &= d_4^+ - d_4^- \\
\boxed{d_1^- = K1}
\end{aligned}$$

Solution Methodology (3rd Obj Function)

Solution of 2nd LP model will yield a value for variable d_2^-
say ; $d_2^- = K2$

Now add the value of this variable in constraint set and solve for third priority variable as follows;

Min d_3^+

subject to

$$\begin{aligned}
7\,X1 + 6\,X2 - 30 &= d_1^+ - d_1^- \\
2\,X1 + 3\,X2 - 12 &= d_2^+ - d_2^- \\
6\,X1 + 5\,X2 - 30 &= d_3^+ - d_3^- \\
X2 - 7 &= d_4^+ - d_4^- \\
d_1^- &= K1 \\
\boxed{d_2^- = K2}
\end{aligned}$$

Solution Methodology (4th Obj Function)

Solution of 3rd LP model will yield a value for variable d_3^+

say ; $d_3^+ = K3$

Now add the value of this variable in constraint set and solve for third priority variable as follows;

Min d_4^-

subject to

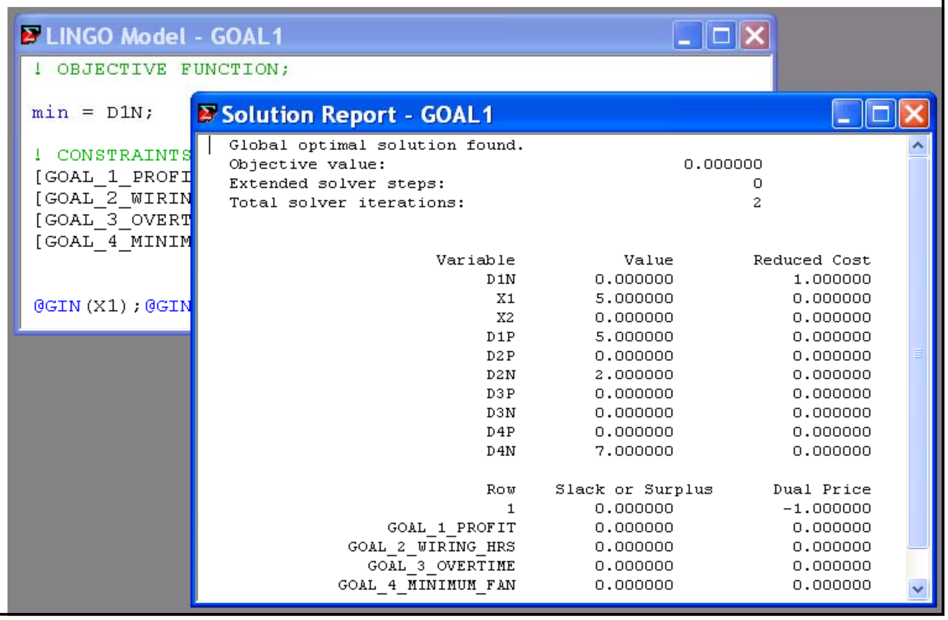
$$
\begin{aligned}
7\,X1 + 6\,X2 - 30 &= d_1^+ - d_1^- \\
2\,X1 + 3\,X2 - 12 &= d_2^+ - d_2^- \\
6\,X1 + 5\,X2 - 30 &= d_3^+ - d_3^- \\
X2 - 7 &= d_4^+ - d_4^- \\
d_1^- &= K1 \\
d_2^- &= K2 \\
\boxed{d_3^+ = K3}
\end{aligned}
$$

APPLYING METHODOLOGY USING LINGO

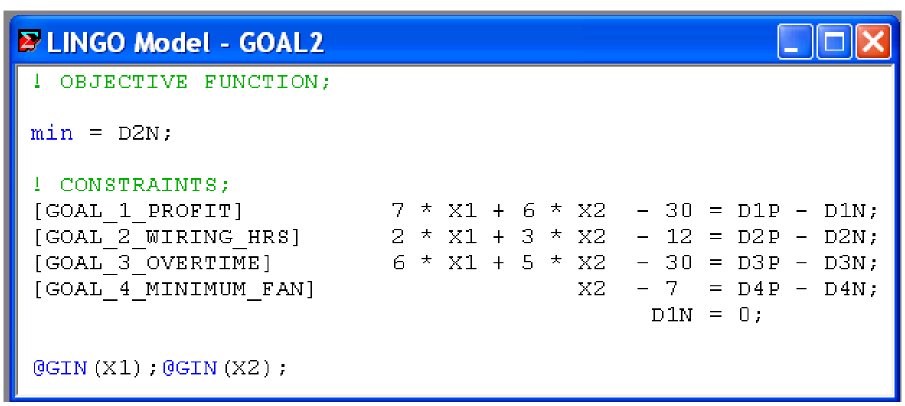
LINGO Model - GOAL1

```
! OBJECTIVE FUNCTION;

min = D1N;

! CONSTRAINTS;
[GOAL_1_PROFIT]          7 * X1 + 6 * X2  - 30 = D1P - D1N;
[GOAL_2_WIRING_HRS]      2 * X1 + 3 * X2  - 12 = D2P - D2N;
[GOAL_3_OVERTIME]        6 * X1 + 5 * X2  - 30 = D3P - D3N;
[GOAL_4_MINIMUM_FAN]              X2  -  7 = D4P - D4N;


@GIN(X1);@GIN(X2);
```

APPLYING METHODOLOGY USING LINGO



APPLYING METHODOLOGY USING LINGO



APPLYING METHODOLOGY USING LINGO

Solution Report - GOAL2

```
Global optimal solution found.
Objective value:                              0.000000
 Extended solver steps:                              0
 Total solver iterations:                            2


                    Variable           Value        Reduced Cost
                         D2N        0.000000            1.000000
                          X1        0.000000            0.000000
                          X2        5.000000            0.000000
                         D1P        0.000000            0.000000
                         D1N        0.000000            0.000000
                         D2P        3.000000            0.000000
                         D3P        0.000000            0.000000
                         D3N        5.000000            0.000000
                         D4P        0.000000            0.000000
                         D4N        2.000000            0.000000

                         Row    Slack or Surplus      Dual Price
                           1        0.000000           -1.000000
               GOAL_1_PROFIT        0.000000            0.000000
           GOAL_2_WIRING_HRS        0.000000            0.000000
             GOAL_3_OVERTIME        0.000000            0.000000
          GOAL_4_MINIMUM_FAN        0.000000            0.000000
                           6        0.000000            0.000000
```

APPLYING METHODOLOGY USING LINGO

LINGO Model - GOAL3

```
! OBJECTIVE FUNCTION;

min = D3P;

! CONSTRAINTS;
[GOAL_1_PROFIT]          7 * X1 + 6 * X2  - 30 = D1P - D1N;
[GOAL_2_WIRING_HRS]      2 * X1 + 3 * X2  - 12 = D2P - D2N;
[GOAL_3_OVERTIME]        6 * X1 + 5 * X2  - 30 = D3P - D3N;
[GOAL_4_MINIMUM_FAN]              X2  - 7  = D4P - D4N;
                                   D1N = 0;
                                   D2N = 0;


@GIN(X1);@GIN(X2);
```

APPLYING METHODOLOGY USING LINGO

Solution Report - GOAL3

```
Objective value:                                 0.000000
Extended solver steps:                                  0
Total solver iterations:                                0
```

Variable	Value	Reduced Cost
D3P	0.000000	1.000000
X1	0.000000	0.000000
X2	6.000000	0.000000
D1P	6.000000	0.000000
D1N	0.000000	0.000000
D2P	6.000000	0.000000
D2N	0.000000	0.000000
D3N	0.000000	0.000000
D4P	0.000000	0.000000
D4N	1.000000	0.000000

Row	Slack or Surplus	Dual Price
1	0.000000	-1.000000
GOAL_1_PROFIT	0.000000	0.000000
GOAL_2_WIRING_HRS	0.000000	0.000000
GOAL_3_OVERTIME	0.000000	0.000000
GOAL_4_MINIMUM_FAN	0.000000	0.000000
6	0.000000	0.000000
7	0.000000	0.000000

APPLYING METHODOLOGY USING LINGO

LINGO Model - GOAL4

```
! OBJECTIVE FUNCTION;

min = D4N;

! CONSTRAINTS;
[GOAL_1_PROFIT]          7 * X1 + 6 * X2  - 30 = D1P - D1N;
[GOAL_2_WIRING_HRS]      2 * X1 + 3 * X2  - 12 = D2P - D2N;
[GOAL_3_OVERTIME]        6 * X1 + 5 * X2  - 30 = D3P - D3N;
[GOAL_4_MINIMUM_FAN]                 X2  - 7  = D4P - D4N;
                                       D1N = 0;
                                       D2N = 0;
                                       D3P = 0;


@GIN(X1);@GIN(X2);
```

APPLYING METHODOLOGY USING LINGO

Solution Report - GOAL4

```
Global optimal solution found.
Objective value:                              1.000000
Extended solver steps:                               0
Total solver iterations:                             0
```

Variable	Value	Reduced Cost
D4N	1.000000	0.000000
X1	0.000000	0.000000
X2	6.000000	-1.000000
D1P	6.000000	0.000000
D1N	0.000000	0.000000
D2P	6.000000	0.000000
D2N	0.000000	0.000000
D3P	0.000000	0.000000
D3N	0.000000	0.000000
D4P	0.000000	1.000000

Row	Slack or Surplus	Dual Price
1	1.000000	-1.000000
GOAL_1_PROFIT	0.000000	0.000000
GOAL_2_WIRING_HRS	0.000000	0.000000
GOAL_3_OVERTIME	0.000000	0.000000
GOAL_4_MINIMUM_FAN	0.000000	-1.000000
6	0.000000	0.000000
7	0.000000	0.000000
8	0.000000	0.000000

Interpretation of Solution

Objective value: 1.000000

D1N	0.000000
D2N	0.000000
D3P	0.000000
D4N	1.000000

X1 = 0 , X2 = 6

OK Profit Deviation ; D1 = D1P – D1N = 6 – 0 = 6
(Profit is $6 more than minimum of $30)

OK Wiring Hours Deviation ; D2 = D2P – D2N = 6 – 0 = 6
(Wiring hours is 6 hours more than minimum of 12)

OK Max Assembly Hours Deviation ; D3 = D3P – D3N = 0 – 0 = 0
(Assembly hours deviation is 0; exactly 30 hours are consumed)

Minimum X2 >= 7 Deviation; D4 = D4P – D4N = 0 – 1 = -1
(One less unit is produced than a minimum requirement of 7 X2 units)

Interpretation of Solution

What is ideal Solution? Obj fn == 0 ***why??***

```
Objective value: 1.000000

D1N          0.000000
D2N          0.000000
D3P          0.000000
D4N          0.000000
```

0

X1 = 0 , X2 = 6

OK Profit Deviation ; D1 = D1P – D1N = 6 – 0 = 6
(Profit is $6 more than minimum of $30)

OK Wiring Hours Deviation ; D2 = D2P – D2N = 6 – 0 = 6
(Wiring hours is 6 hours more than minimum of 12)

OK Max Assembly Hours Deviation ; D3 = D3P – D3N = 0 – 0 = 0
(Assembly hours deviation is 0; exactly 30 hours are consumed)

Minimum X2 >= 7 Deviation; D4 = D4P – D4N = 0 – 1 = -1
(One less unit is produced than a minimum requirement of 7 X2 units)

Goal Programming Versus Linear Programming

- Multiple goals (instead of one goal)
- Deviational variables minimized (instead of maximizing profit or minimizing cost of LP)
- “Satisficing” (instead of optimizing)
- Deviational variables are real (and replace slack variables)